Integration by parts

$$\boxed{\int u\,dv = uv - \int v\,du}$$

<u>MORAL</u> When choosing u and dv,

u - function that becomes simpler when differentiated

dv = which can be easily integrated to get v.

$$\int \ln x \, dx$$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$dv = dx \qquad v = x$

$$x \ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int dx = x\ln x - x + C$$

$$\int x^2 e^x \, dx$$

$u = x^2 \qquad du = 2x \, dx$

$dv = e^x \, dx \qquad v = e^x$

$$\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x dx$$

$$= x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2 \left[x e^x - e^x + C \right]$$

$$= x^2 e^x - 2x e^x + 2e^x + D.$$

$$\int x e^x \, dx = ?$$

$u = x \qquad du = dx$

$dv = e^x \, dx \qquad v = e^x$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

Ex $\int e^{3x}\cos x\,dx$ → neither simplifies when we take derivatives.

Let $u = e^{3x}$ $\quad du = 3e^{3x}\,dx$

$dv = \cos x\,dx$ $\quad v = \sin x$

$$\int e^{3x}\cos x\,dx = e^{3x}\sin x - \int \sin x \cdot 3e^{3x}\,dx$$

$$= e^{3x}\sin x - 3\int e^{3x}\cdot\sin x\,dx$$

$$= e^{3x}\sin x - 3\left[e^{3x}\cdot(-\cos x) - \int(-\cos x)3e^{3x}\,dx\right]$$

↳ $u = e^{3x}$; $dv = \sin x\,dx$

$du = 3e^{3x}\,dx$; $v = -\cos x$

$$\int e^{3x}\cos x\,dx = e^{3x}\sin x + 3e^{3x}\cos x - 9\int e^{3x}\cos x\,dx$$

$$10\int e^{3x}\cos x\,dx = e^{3x}\sin x + 3e^{3x}\cos x$$

$$\Rightarrow \int e^{3x}\cos x\,dx = \frac{1}{10}\left[e^{3x}\sin x + 3e^{3x}\cos x\right] + C$$

If we have definite integrals

$$\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du .$$

<u>Ex</u> Calculate $\displaystyle\int_0^1 \tan^{-1}x \, dx$

Let $u = \tan^{-1}x$ $\qquad$ $dv = dx$

$du = \dfrac{1}{1+x^2}\,dx$ $\qquad$ $v = x$

$$\int_0^1 \tan^{-1}x\,dx = x\tan^{-1}x\Big]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= 1 \cdot \tan^{-1}(1) - 0 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}\,dx$$

Let $u = 1 + x^2$ $\qquad x = 0,\ u = 1$

$du = 2x\,dx$ $\qquad x = 1,\ u = 2$

$$dx = \frac{du}{2x}$$

$$= \frac{\pi}{4} - \int_1^2 \frac{x}{u} \frac{du}{2x}$$

$$= \frac{\pi}{4} - \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln|u| \Big]_1^2$$

$$= \frac{\pi}{4} - \frac{1}{2}\left[\ln 2 - \ln 1\right]$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$